…HEMATICS 30&33

…UIDE 1982

…riculum

Alberta
EDUCATION

ACKNOWLEDGEMENT

Alberta Education acknowledges with appreciation the contribution of the following members of the Committee on Senior High School Mathematics operating under the direction of the Mathematics Coordinating Committee and the Curriculum Policies Board.

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MATHEMATICS 30 AND 33
INTERIM CURRICULUM GUIDE

PLEASE NOTE:

This curriculum guide is interim in nature. The guidelines presented in this curriculum guide will be subject to revision according to information received from schools during the 1982-83 school year.

A comprehensive curriculum guide and resource guide will be produced when the senior high school mathematics program is completed.

The full program will be implemented as follows:

	OPTIONAL	MANDATORY
Math 10, 13	September, 1980	September, 1981
Math 20, 23	September, 1981	September, 1982
Math 30, 33	September, 1982	September, 1983

NOTE: THIS PUBLICATION IS A SERVICE DOCUMENT. The advice and direction offered is suggestive except where it duplicates or paraphrases the contents of the Program of Studies.

ORIENTATION TO THE PROGRAM

A. STRUCTURE

Core

A common set of mathematics objectives has been identified as basic to both the Mathematics 30 and Mathematics 33 course. The topics of this core program are a continuation of those presented in the grade 11 program.

Trigonometry and Statistics are new to the grade 10 program. These topics are further developed as complete strands through the remaining two high school grades.

It should be noted that even though some of the objectives are similar for the two courses the methods of instruction and the time required for topics will not be the same. An experimental approach with more time being devoted to basic skills is suggested for the Mathematics 33 program.

Independent Core

While common objectives of topics have been identified as core, each course has a separate set of objectives specific to the course. These objectives are listed under the title of independent core.

The Mathematics 30 independent core goes further into the theoretical development of mathematics concepts with a greater emphasis on formalization of thought.

The model of instruction suggested for the Mathematics 33 independent core would be based on an experimental/inductive approach with an increase in time spent on exploratory activities.

Electives

Elective topics in the senior high school mathematics program have been included only at the grade 11 and 12 levels. The grade 10 program does not have an elective component to ensure sufficient time for an adequate mathematical foundation for subsequent courses.

B. AREAS OF EMPHASIS

Course Content

A deliberate attempt has been made to reduce course content with possible gains in the form of learning by discovery. The number of concepts and the depth of coverage has been reduced in an attempt to provide time for teachers and students to pursue different interesting ideas.

Problem Solving

A deliberate attempt has been made to stress problem solving. This is a crucial aspect of mathematical activity and will require additional activities, and planning on the part of teachers and students.

Application

A deliberate attempt has been made to relate mathematics to real life situations with a view to making the study of mathematics more meaningful to the vast majority of students.

Resource Material

A section of comments and possible applications has been prepared to give some suggestions to teachers. This section will be improved as more ideas are supplied by teachers. A comprehensive resource guide including references, materials, films and other aids will be produced as the complete high school program is developed.

MATHEMATICS PROGRAM OVERVIEW

ELECTIVE	INDEPENDENT CORE	CORE	INDEPENDENT CORE	ELECTIVE
Matrices Math Induction Trigonometric Identities Binomial Theorem Polynomial Functions Arrangements and Selections Transformations (Conics) Complex Numbers Transformational Geometry History of Math Math Art Computer Literacy Absolute Value Topology Probability Linear Programming Vectors Inequalities Consumer Mathematics Area and Volume Mathematics of Accounting Industrial Mathematics	**MATH 30** Sequences, Series and Limits Trigonometry Quadratic Relations (Conics)	Presentation of Data and Descriptive Statistics Logarithms Trigonometry	**MATH 33** Relations and Functions Quadratic Functions, Equations and Applications	History of Math Arrangements and Selections Math Art Computer Literacy Topology Probability Linear Programming Vectors Inequalities Consumer Mathematics Area and Volume Mathematics of Accounting Industrial Mathematics Complex Numbers Transformational Geometry Matrices
	MATH 20 Relations and Functions Polynomials Systems of Equations Quadratic Functions, Equations and Applications Radicals Geometry	Coordinate Geometry Systems of Equations Radicals Variation Trigonometry Geometry Presentation of Data Descriptive Statistics Polynomials	**MATH 23** Radicals Geometry Polynomials	
	MATH 10 Number Systems Exponents and Radicals Geometry (Deductive) Polynomials Equations and Graphing	Number Systems Variation Exponents Equations and Graphing Geometry Polynomials Trigonometry Presentation of Data and Descriptive Statistics	**MATH 13** Geometry (Inductive)	

(Revised September 1982)

SUGGESTED TIME ALLOCATIONS

Math 30

Topic	Number of Hours
A. Trigonometry	28
B. Quadratic Relations	25
C. Sequences, Series, Limits	20
D. Statistics	18
E. Logarithms	10
F. Electives	24
	125

Math 33

Topic	Number of Hours
A. Relations and Functions	15
B. Trigonometry	20
C. Statistics	21
D. Quadratic Functions, Equations	30
E. Logarithms	15
F. Electives	24
	125

TEXTS AND REFERENCES

MATHEMATICS 30:

Prescribed References: *Math Is/6*

Frank Ebos, Bob Tuck
Nelson/Canada Ltd., 1982

Foundations of Mathematics for Tomorrow: Senior

Dino Dottori, George Knill, James Stewart
McGraw-Hill Ryerson Ltd., 1979

Recommended Reference: *Holt Mathematics 6*

Kenneth D. Fryer, Ronald G. Dunkley,
H. A. Elliott, Norman J. Hill, R. Jock MacKay
Holt, Rinehart and Winston of Canada Ltd., 1981

MATHEMATICS 33:

Prescribed Reference: *Applied Mathematics for Today: Senior*

Book 1 Second Edition
Dino Dottori, George Knill, John Seymour
McGraw-Hill Ryerson Ltd., 1977

Recommended Reference: *Mathematics For a Modern World*

Book 4
E. G. Carli, J. C. Egsgard, C. Psica, J. J. Del Grande
Gage Publishing Ltd., 1975

LEARNING RESOURCE APPROVALS

In terms of provincial policy, learning resources are those print, nonprint and electronic courseware materials used by teachers or students to facilitate teaching and learning.

PRESCRIBED LEARNING RESOURCES are those learning resources approved by the Minister as being most appropriate for meeting the majority of goals and objectives for courses, or substantial components of courses, outlined in provincial Programs of Study.

RECOMMENDED LEARNING RESOURCES are those learning resources approved by Alberta Education because they complement Prescribed Learning Resources by making an important contribution to the attainment of one or more of the major goals of courses outlined in the provincial Programs of Study.

SUPPLEMENTARY LEARNING RESOURCES are those additional learning resources identified by teachers, school boards or Alberta Education to support courses outlined in the provincial Programs of Study by reinforcing or enriching the learning experience.

GOALS OF THE SENIOR HIGH SCHOOL MATHEMATICS PROGRAM

Although the different courses of the senior high school mathematics program have different specific objectives, the goals of the senior high mathematics program are set forth in relation to three main expectations and needs: those of the individual, those of the discipline of mathematics and those of society at large. They are listed as follows:

Student Development

a) To develop in each student a positive attitude towards mathematics.

b) To develop an appreciation of the contribution of mathematics to the progress of civilization.

c) To develop the ability to utilize mathematical concepts, skills and processes.

d) To develop the powers of logical analysis and inquiry.

e) To develop an ability to communicate mathematical ideas clearly and correctly to others.

Discipline of Mathematics

a) To provide an understanding that mathematics is a language using carefully defined terms and concise symbolic representations.

b) To provide an understanding of the concepts, skills and processes of mathematics.

c) To provide an understanding of the common unifying structure in mathematics.

d) To furnish a mode of reasoning and problem solving with a capability of using mathematics and mathematical reasoning in practical situations.

Societal Needs

a) To develop a mathematical competence in students in order to function as citizens in today's society.

b) To develop an appreciation of the importance and relevance of mathematics as part of the cultural heritage that assists people to utilize relationships that influence their environment.

c) To develop an appreciation of the role of mathematics in man's total environment.

PROBLEM SOLVING

The discipline of mathematics provides one of the best opportunities to practise the process of problem solving. The techniques learned and experience gained are transferable to almost any problem solving situation and provide the basis of successful living.

Everyone's job involves problem solving. Although many of the problems we have to contend with are repetitious and are easily resolved once the initial solution is known, the most challenging and exciting aspects deal with problems that are completely new to the individual or assist in arriving at solutions in new and creative ways.

A primary purpose of education is to provide a student with the tools to solve a wide variety of problems. Specific problem solving skills can be taught; however, the methods of successfully tackling untried problems is not always fully understood. There is a sequence of steps by which one can proceed, such as:

a) fully understanding the problem

b) accepting the challenge of searching for a solution

c) making a reasonable guess at the answer (this provides a check when a solution is finally found)

d) pictorially or graphically representing the important aspects of the problem if this is at all possible

e) solving a simpler version of the problem or a special case of the more general problem

f) using specific cases to discover patterns or different approaches to the problem

g) solving the general problem

h) checking the solution in the special cases solved before (Does the solution seem reasonable based on your initial guess?)

The above sequence of steps is helpful in tackling any problem. Considerable intuition and experience are still necessary to tackle the really difficult problems.

STATISTICS

Statistics is an exciting new addition to the mathematics curriculum. It provides many opportunities for applications and hence is easily motivating.

Statistics, over the past number of years, has played an increasing role in public information. We are continually bombarded by statistical information, such as Consumer Price Index, baseball averages, weather forecasting, election polls and stock market indices.

Statistics is being used extensively in research and industry. In any enterprise where large amounts of data need to be handled and processed, where predictions are to be made, statistical methods play a role. Statistical techniques are also used in designing telephone exchanges, optimally setting traffic lights, designing insurance policies, determining the reliability of one's car, criminal detection, and market research, to mention a few.

The introduction of statistics at this level is intended to familiarize the student with the elementary descriptive measures that form the basis of any further work with large data sets. In most cases the data should be collected by the student, preferably different data for each student. This could be a stimulating project. The subsequent analysis then becomes more meaningful and hopefully provides not only statistical insight, but some new knowledge about our surroundings. In order to understand and intelligently discuss much of the information to which we are daily subjected, some knowledge of the terminology and underlying assumptions of statistics is necessary.

Statistics is an applied science. To motivate this subject, especially at the introductory level, one should concentrate on the experimental approach. This unit provides the opportunity for field trips to collect data. By displaying this data in various forms via graphs, one can make some basic inferences about the data source. To justify the collection of the data, it is important to make some inference. Just as important is to ask what further information would be worth knowing and discuss how one would proceed to discover it.

APPLICATIONS

A central and encompassing approach to making the study of mathematics both meaningful and interesting is to place a major emphasis in the area of applications. An application is the process of using a mathematical skill to arrive at a solution to a real life or practical situation. Applications should incorporate interesting, useful, relevant and diverse examples from real life situations. The significance of using applications is based on relating mathematical concepts and skills to problems encountered in society and the environment. Mathematics may be related to countless aspects of living. Our task is to have students recognize these relations, develop an understanding of the interrelationships that exist with a mathematics program and then learn to transfer their present use of mathematics to other situations.

Applications, like problem solving, should be integrated into the overall program rather than be dealt with as an independent unit. Whenever possible, integration and coordination with other subject areas should be encouraged. When applications require extensive computations, the use of the calculator may become a necessary component of the learning process to avoid time-consuming calculations.

THE ELECTIVE COMPONENT

The elective component is an integral part of the senior high school program at the grade 11 and 12 levels. The elective component primarily offers an opportunity for students to spend time on interesting and useful areas of mathematics not necessarily contained in the core and independent core component.

The time allotment for the elective component should be approximately 24 hours at the grade 12 level. It is suggested that the 24 hours of electives be determined according to the plans of the individual teacher.

A. Structure of the Elective

The elective component of the program may be:

1) a content area not prescribed as a core or independent core topic

OR

2) a locally developed unit as determined by the teacher or school system

OR

3) an extension of the subject matter in any of the core or independent core topics to provide students with enrichment.

B. Guidelines

Teachers should keep the following guidelines in mind:

1) The topics are open-ended so that the interests and abilities of students may be taken into account.

2) Student initiated projects may be considered as an elective.

3) A teacher should make use of any appropriate resources.

4) Electives should be included in the course throughout the year, wherever appropriate. These should not necessarily be taught during a 24 hour block. Where more than one elective is included, the time for each elective does not necessarily have to be the same.

5) The elective component of the program should be included in the evaluation of the students.

6) Prerequisite core material may be required before some electives are attempted.

C. Suggested Time Allotments

The time allocated to topics in the elective component of the course may vary according to the topic(s) chosen and instructional preferences of the teacher. A total of 24 hours may be devoted to cover one elective topic, or alternatively, two or more topics may be covered. The time allotted to the elective topic is at the discretion of the teacher. Total time for electives is 24 hours.

Example 1: Computer Literacy covering all 24 hours.

Example 2: Consumer Mathematics/Trigonometric Identities covering 24 hours.

Example 3: Vectors/Math Art/Inequalities covering 24 hours.

D. Suggested Elective Outlines

The elective component is designed to be an interesting and motivational aspect of the mathematics program. The outlines (on pages 59 to 94) for elective topics are intended to act as guidelines for individual teachers. Teachers may wish to follow the suggested outlines or incorporate their own ideas for the elective component. The prescribed references provide useful material for many of the elective units.

E. Provision for the Academically Talented

Topics within the elective component of the course should be utilized to challenge the academically talented students. Such topics as computer literacy and topology, for example, may be used to provide challenge to students of this calibre. Teachers may also extend core topics and concepts to a higher level of complexity to meet the needs of the stronger academic student.

MATHEMATICS 30 AND 33
COURSE OBJECTIVES

MATHEMATICS 30 AND MATHEMATICS 33

A. RELATIONS AND FUNCTIONS

MATH 30	MATH 33	COMMON CORE	
			1. Define a relation and its domain and range, utilizing graphs and algebraic statements.
			2. Define the term function.
			3. Use functional notation f(x) for particular values of x.
			4. Define and graph a linear function.
			5. Show the relationship between the graphs of linear and quadratic functions and the roots of the corresponding equations.
			B. TRIGONOMETRY
			1. Maintain previously developed skills.
			2. Describe circular paths using the initial point and directed distance of the path.
			3. Define the unit circle $x^2 + y^2 = 1$.
			4. Determine coordinates of points on the unit circle.
			5. Define the trigonometric ratios in terms of coordinates of points on the unit circle.

MATH 30	MATH 33	COMMON CORE	
			6. Find the domain and range of the six trigonometric functions.
			7. Solve simple trigonometric equations.
			8. Given the value of one of the trigonometric ratios, evaluate the other five trigonometric ratios.
			9. Derive and apply the following identities: a. Quotient relations: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$ b. Reciprocal Relations: $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$ c. Pythagorean Relations: $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$
			10. Derive and apply the following identities: a. Negative Arc Formulas: $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

MATH 30	MATH 33	COMMON CORE	
			10. Continued. b. Sum Formulas: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ c. Complementary Arc Formulas: $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
			11. Draw and identify the graphs of the sine, cosine and tangent functions.
			12. Define periodic function and state the periods of $\sin\theta$, $\cos\theta$, $\tan\theta$.
			13. Define radian measure.
			14. Convert degree measure to radian measure and vice versa.
			15. Determine exact values of trigonometric ratios of 0°, 30°, 45°, 60° and 90°.
			16. Determine the value of a trigonometric ratio of any angle.
			17. Solve oblique triangles by using the sine law and/or cosine law.
			18. Apply the sine law and cosine law to practical problems.
			19. Solve problems involving area of regular polygons.

C. QUADRATIC RELATIONS (CONIC SECTIONS)

MATH 30	MATH 33	COMMON CORE	
			1. Maintain previous skills in analytic geometry: a. linear functions and slope b. distance and midpoint formula c. properties of tangents from Math 20 d. solution of systems of equations in two variables
			2. State the definition of the circle and derive the standard form.
			3. Convert the equation of a circle from standard to general form.
			4. Determine the equation of a circle and sketch the graph given these conditions: a. centre and radius b. centre and a point c. centre and equation of tangent line d. three points on a circle e. two points and equation of line containing the centre
			5. Define and identify a parabola and the terms: focus, vertex and axis.
			6. Derive the standard form of the equation of a parabola with horizontal or vertical axis of symmetry.
			7. Find the focus and direction from the equation of a parabola.
			8. Determine an equation and sketch the parabola given: a. focus and directrix b. vertex and directrix c. vertex and point on the parabola

MATH 30	MATH 33	COMMON CORE	
			9. Solve applied problems related to the parabola.
			10. Define and identify an ellipse and the terms: foci, major axis, minor axis, vertices and focal radii.
			11. Derive the standard form of the equation of an ellipse with foci on the x-axis or y-axis.
			12. Given the equation of the ellipse, determine: foci, vertices, major axis and minor axis.
			13. Derive the relation between the parameters a, b and c for the ellipse. $(a^2 = b^2 + c^2)$
			14. Determine an equation and sketch the ellipse given: a. minor axis and distance between foci b. vertices and foci c. vertices and a point on ellipse
			15. Solve applied problems related to the ellipse.
			16. Define and identify a hyperbola and the terms: vertices, foci transverse axis, conjugate axis and asymptotes.
			17. Derive the standard form of the equation of a hyperbola with foci on the x-axis or the y-axis.
			18. From the equation of the hyperbola find the foci, vertices, transverse axis, conjugate axis and asymptotes.

MATH 30	MATH 33	COMMON CORE	
			19. Derive the relation between the parameters a, b and c for the hyperbola. $(a^2 + b^2 = c^2)$
			20. Determine an equation and sketch the hyperbola given: a. transverse axis and conjugate axis b. foci and length of one axis c. equation of an asymptote and a point on the hyperbola
			21. Solve applied problems related to the hyperbola.

D. SEQUENCES, SERIES, LIMITS			
			1. Recognize the difference between: a. a sequence and a series b. finite and infinite sequences
			2. a. Recognize and define arithmetic sequences and series and state the common difference (d) b. Derive and apply: i. the general term formula $a_n = a_1 + (n-1)d$ ii. the sum formula $S_n = \frac{n}{2}(a_1 + a_n)$ c. Apply formulas to problems involving arithmetic sequences and series.
			3. a. Recognize and define geometric sequences and series and state the common ratio (r)

MATH 30	MATH 33	COMMON CORE	
			3. Continued. b. Derive and apply: i. the general term formula $a_n = a_1 r^{n-1}$ ii. the sum formulas $S_n = \frac{a_1(r^n-1)}{r-1}, r \neq 1$ $S_n = \frac{ra_n - a_1}{r-1}, r \neq 1$ c. Apply formulas to problems involving geometric sequences and series, with special emphasis being given to the mathematics of finance: i. Difference between and applications of simple and compound interest. ii. Use tables to determine accumulated and present value accounts involving compound interest over different time periods. iii. Illustrate the various annuities by using line diagrams. iv. Apply geometric series to both accumulated and present value annuities with both identical and differing interest and payment periods.
			4. Generate the terms of a series using sigma notation (Σ).
			5. Determine the limits of various functions.
			6. Recognize the differences between infinite convergent and divergent sequences.
			7. Find the limits of infinite convergent sequences.
			8. Find the sums of infinite convergent series.

MATH 30	MATH 33	COMMON CORE	
			9. Solve problems involving infinite geometric series.
			E. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS
			1. Maintain previous skills. a. frequency distribution b. measures of central tendency c. measures of dispersion
			2. Illustrate and develop the normal distribution.
			3. Develop and apply standard deviation.
			4. Introduce probability using an experimental approach.
			5. Apply probability to theoretical frequency distribution.
			F. QUADRATIC FUNCTIONS, EQUATIONS AND APPLICATIONS
			1. Identify and express quadratic functions in the form $y = ax^2 + bx + c$ where $a, b, c \in R$, $a \neq 0$
			2. Identify and express quadratic equations in the form $ax^2 + bx + c = 0$ where $a, b, c \in R$, $a \neq 0$
			3. Graph a quadratic function using a table of values.
			4. Find the vertex, axis of symmetry, domain, range and maximum or minimum value of a quadratic function from its graph.

MATH 30	MATH 33	COMMON CORE	
			5. Use the formula for vertex and axis of symmetry if the quadratic function is given in the form $y = ax^2 + bx + c$.
			6. State the relationship between the graph of a quadratic function and the roots of the corresponding equation.
			7. Write quadratic equations in the form $ax^2 + bx + c = 0$ and specify the value of a, b, c.
			8. Use the method of completing the square of a quadratic function to find the vertex, axis of symmetry, range and maximum or minimum value. Draw the graph using this information.
			9. Solve problems involving the maximum or minimum value of a quadratic function.
			10. Compute the real roots of a quadratic equation by: a. factoring b. using the quadratic formula or completing the square
			11. Define and evaluate the discriminant of a quadratic equation.
			12. State the nature of the roots by examining the discriminant.
			13. Solve problems whose solutions are based on quadratic equations.

G. LOGARITHMS

MATH 30	MATH 33	COMMON CORE	
			1. Maintain previous skills on exponents.
			2. Identify and graph exponential functions.
			3. Convert equation from exponential form to logarithmic form and vice-versa.
			4. Solve logarithmic equations by converting to exponential form.
			5. Define the inverse of an exponential function in logarithmic form.
			6. Evaluate expressions and solve equations involving logarithmic form and exponential form.
			7. State and use the basic laws or properties of logarithms, products, quotients, powers and roots.
			8. Use logarithms to solve practical problems.

MATHEMATICS 30:
PRIMARY REFERENCE CORRELATION

MATHEMATICS 30: PRIMARY REFERENCE CORRELATION

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
A. TRIGONOMETRY				
1. Maintain previously developed skills.	Review terms associated with rectangular coordinate system; review the theorem of Pythagoras. Find coordinates of points on the x-axis and y-axis. Note: The relationship between degrees, minutes and decimal degrees should be discussed.		P.7 P.251-255 P.209-210	P.175-182
2. Describe circular paths using the initial point and directed distance of the path.			P.203-207	P.208-212
3. Define the unit circle $x^2 + y^2 = 1$.	The unit circle can be the basis for definition of trigonometric functions; if (x,y) is the terminal point of the circular arc determined by Θ, then $x = \cos \Theta$ and $y = \sin \Theta$.		P.213,248	
4. Determine coordinates of points on the unit circle.	A good starting place would be to consider the points on the unit circle where the x-axis and the y-axis intersect the unit circle. Continue by determining coordinates of points corresponding to angles which are multiples of 30 degrees and 45 degrees.		P.213-216 P.248	
5. Define the trigonometric ratios in terms of coordinates of points on the unit circle.	Exemplify by determining exact values of the trigonometric ratios for angles which are multiples of 30 degrees and 45 degrees.		P.213	

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
6. Find the domain and range of the six trigonometric ratios.	These concepts are useful when discussing the amplitude of a trigonometric function graph. For example, $y = \sin \Theta$ has a range $-1 \leqq \sin \Theta \leqq 1$. Thus its amplitude is 1. For $y = 3 \sin \theta$, the range is $-3 \leqq \sin \theta \leqq 3$. Thus its amplitude is 3.		P.226	P.212-224
7. Solve simple trigonometric equations.	Coverage should include equations of the linear and quadratic format. For example, $3 \cos \Theta + 1 = 2$, $2 \sin^2\Theta + \sin \Theta - 1$ and $2 \tan^2 2\Theta - 3 \tan 2\Theta - 1 = \Theta$. Note that the same argument (e.g. Θ, 2Θ, $\frac{\Theta}{2}$) should be used throughout the equation.		P.219 P.242-246	P.235-238
8. Given the value of one of the trigonometric ratios, evaluate the other five trigonometric ratios.	Consider both exact and approximate values.		P.211-216	P.231-235
9. Derive and apply the following identities: a. Quotient relations: $\tan \Theta = \frac{\sin \Theta}{\cos \Theta}$, $\cot \Theta = \frac{\cos \Theta}{\sin \Theta}$			P.239-243	P.231-232

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OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
9. (b) Reciprocal relations: (cont) $\csc \Theta = \frac{1}{\sin \Theta}$, $\sec \Theta = \frac{1}{\cos \Theta}$ $\cot \Theta = \frac{1}{\tan \Theta}$ (c) Pythagorean relations: $\sin^2\Theta + \cos^2\Theta = 1$ $1 + \tan^2\Theta = \sec^2\Theta$ $1 + \cot^2\Theta - \csc^2\Theta$			P.239-243	P.231-232
10. Derive and apply the following identities: a. Negative arc formulas: $\sin(-\Theta) = -\sin\Theta$ $\cos(-\Theta) = \cos\Theta$ $\tan(-\Theta) = -\tan\Theta$ b. Sum formulas: $\sin(A \pm B) \quad \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ c. Complementary arc formulas: $\cos\left(\frac{\pi}{2} - \Theta\right) = \sin\Theta$ $\sin\left(\frac{\pi}{2} - \Theta\right) = \cos\Theta$	Identities involving extremely long solutions should be avoided. Most identities require transformation of the side with the more complicated expression into a new form which is the same as that appearing on the other side. In some cases, both sides will have to be transformed into the same expression.		P.220 P.413-415	P.242-243 P.179-183

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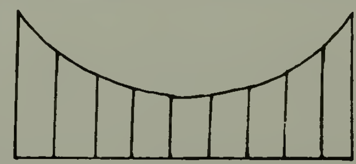
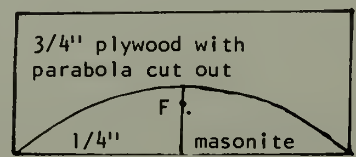
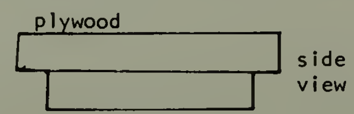
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
11. Draw and identify graphs of the sine, cosine and tangent functions.	Consider the effects of parameters a, b and c on the graph $Y = a \sin (bx) + c$. Effects of changes in these parameters can be considered individually, and then in combination. Concepts of phase shift, amplitude and period should be introduced.	Students can chart their own bio-rhythms using sine curves. This is highly motivational application of trigonometric function graphs. See *Mathematics Teacher*, October, 1977. Amplitude, period and frequency of sine waves can be demonstrated using an audio generator and an oscilloscope. Assistance from your physics department may be required. Periodic functions which are non-trigonometric such as square-wave and sawtooth may also be demonstrated if appropriate equipment is available. Note: several computer graphing experiments are becoming available.	P.226-231	P.212-217
12. Define periodic function and state the periods of $\sin \theta$, $\cos \theta$ and $\tan \theta$.			P. 226	P.214
13.. Define radian measure.		Radian measure can be used to find the length of an arc along a great circle of a sphere (e.g., the earth) by the formula $S = r\theta$. This formula requires θ to be a radian measure. Radian measure is frequently used to describe angular velocity of a rotating pulley or shaft, such as is found in farm machinery or cars.	P.205-206	P.208-212

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
14. Convert degree measure to radian measure and vice versa.	Conversion formulas depend on the fact that an arc of a circle is proportional to the central angle which it intercepts. For example, a semi-circle intercepts an angle of 180 degrees.		P.206-207	P.208-212
15. Determine exact values of trigonometric ratios of $0°$, $30°$, $45°$, $60°$ and $90°$.	A number such as $\sqrt{2}$ is exact. This is a difficult concept for students who will read the decimal equivalent of 1.414 and say that it is more exact. It should be emphasized that the decimal is infinite and has been rounded off in the tables.		P.212-213	P.244-245
16. Determine the value of a trigonometric ratio of any angle.	This may be developed easily using the addition formulas for sin $(A + B)$. For example, $\sin 140° = \sin (90° + 50°)$. Alternatively related angles may be illustrated by geometric ideas such as congruent triangles or reflections. In actual practice students will find it easier to use the 3-step reduction of angles method. 1. Find the related angle. 2. Use the same function as the given one. 3. Determine the sign of the function.		P.216-220 P.223-225	P.182-183
17. Solve oblique triangles by using the sine law and/or cosine law.	Law of sines: Case II - Problems in which two sides and an angle opposite one of them are given may have more than one solution depending on the relative measure of the sides (ambiguous case). Better students may wish to pursue an analysis of this case in detail.		P.257-267	P.183-195

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
18. Apply the sine law and cosine law to practical problems.		In aerial navigation, determine which course to fly to counteract the effects of wind. In surveying, find inaccessible distances.	P.257-267 P.274-277	P.196-199 P.200-204
19. Solve problems involving areas of regular polygons.	Find the length of a side of a regular polygon inscribed in a circle of known radius; also find the length of the segment from the centre to the midpoint of any side. These quantities can then be used to find the area of the polygon.	Find dimensions of a hexagonal bolt head, the radius of circular rod from which it must be cut and the length of each side of the head.	P.412	

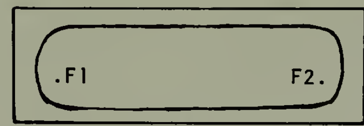
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
B. QUADRATIC RELATIONS (CONIC SECTIONS)				
1. Maintain previous skills in analytic geometry: a. linear functions and slope b. distance and midpoint formulas c. properties of tangents (from Math 20) d. solution of systems of equations in two variables	In addition to 2-variable systems, it is worth extending this concept to three-equations with three unknowns. This will be required in some problems involving the equation of a circle. The same techniques of elimination of variables by addition-subtraction or substitution can be used.		P.X-XII P.347 P.360-365	1.a. P402 b. P402 c. P392-396 d. P425
2. State the definition of the circle and derive the standard form.	The definition involves the concept of locus (path traced by a point moving according to a certain rule). Formulas should be derived for the circle with centre at the origin and also at some other point off the origin.		P.370-374	P.397-401
3. Convert the equation of a circle from standard to general form.	When this conversion has been completed, students should be able to find the centre and radius by using the formulas.		P.374-375 P.378	P.397-401
4. Determine the equation of a circle and sketch the graph given these conditions: a. centre and radius b. centre and a point c. centre and equation of tangent line d. three points on a circle e. two points and equation of line containing the centre	These problems require one of several approaches depending on the given information: a. direct substitution of centre and radius into standard form b. calculation of radius using distance formula c. use of tangent at point of contact d. substitution of coordinates of points into the general form and solving the resulting system of equations. Care should be taken in assigning problems of type 4(e) as these can be very long and discouraging to students.		P.375-380 P.402-404	a.P.401 b.P.401

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
5. Define and identify a parabola and the terms: focus, vertex, axis.			P.381	P.404-406
6. Derive the standard form of the equation of a parabola with a horizontal or vertical axis of symmetry.	Use the "focus-directrix" definition of a parabola and the distance formula. There are four equations required depending on whether the parabola opens upward, downward, to the left or to the right. This should include parabolas whose vertices are off the origin.		P.382	P.404-405
7. Find the focus and directrix from the equation of a parabola.	Use the formula derived in objective 6 to determine the orientation of the parabola. By comparing the given equation to one of the four types, the value of p can be determined. $y^2 = -32x$ can be compared to $y^2 = -4px$, hence $p = 8$.		P.382-389 P.406	P.404-406
8. Determine an equation and sketch the parabola, given: a. focus and directrix b. vertex and directrix c. vertex and point on the parabola	Determine the orientation of the parabola to find which of the four equations should be used. Find the value of p, either from the diagram or by substitution of coordinates of a known point into the formula.		P.382-384	b.P.406
9. Solve applied problems related to the parabola.		1. Parabolic reflectors for sound (microphone at the focus) or micro-wave (antenna at the focus). 2. Cut away view of an auto headlight.	P.385	P.419-424

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
9. continued		3. Suspension bridge supported by a cable in the form of a parabola.  Find the length of one of the cables supporting the roadway. 4. Parabolic pool table.   To demonstrate the reflection property of a parabola place a rubber ball at F. Using a pointer (pool cue) shoot another ball along any line parallel to the main axis. When the ball bounces		

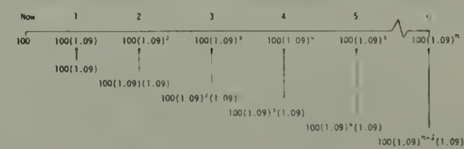
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
9. continued		off the parabola, it will hit the ball placed at F. Conversely, the ball placed at F, when shot towards the parabola, will travel outward along a parallel line. 5. Height of a parabolic arch over a roadway.		
10. Define and identify an ellipse and the terms: foci, major axis, minor axis, vertices, and focal radii.			P.386	P.406-411
11. Derive the standard form of the equation of an ellipse with foci on the x-axis or y-axis.	Use the "constant sum" definition and distance formula to derive the equation. The meaning of the parameters a, b, and c should be clearly demonstrated on a graph before the derivation is attempted.		P.386-387	P.406-411
12. Given the equation of the ellipse determine: foci, vertices, major axis and minor axis.	Students should recognize the form of the equation and hence the orientation of the graph. The parameters a, b, and c can then be determined.		P.388-389	P.406-411
13. Derive the relation between the parameters a, b and c for the ellipse. $(a^2 = b^2 + c^2)$			P.387	P.406-411

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
14. Determine an equation and sketch the ellipse given: a. minor axis and distance between foci b. vertices and foci c. vertices and a point on the ellipse	A rough sketch can help determine the required parameters. In some cases, substitution of the coordinates of a point on the ellipse is required. Ellipses with centre off the origin are not included.		P.388-389	b.P.411
15. Solve applied problems related to the ellipse.		1. Orbital path of a Satellite. 2. Whispering Galleries - an elliptical shaped room with two listening posts (one at each focus). A person whispering at one focus will be heard by a second person at the other focus because of the reflective properties of an ellipse. A person standing between the two foci will not hear the conversation. 3. Eccentric Gears - one turns at a constant rate while the other has a varying speed of rotation. An elliptical shaped cam in a car engine operates in a similar way.	P.390	P.419-424

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
15. continued		4. <u>An Elliptical Pool Table</u> to demonstrate the reflective properties of an ellipse.  3/4" plywood A ball starting at F1 will reflect off the ellipse and pass through F2.		
16. Define and identify a hyperbola and the terms: vertices, foci, transverse axis, conjugate axis and asymptotes.			P.391	P.412-418
17. Derive the standard form of the equation of a hyperbola with foci on the x-axis or the y-axis.	Use the "constant difference" definition and distance formula to derive the equation. The meaning of parameters a, b and c should be clearly demonstrated before the derivation is attempted.		P.391-392	P.414

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
18. From the equation of the hyperbola find the foci, vertices, transverse axis, conjugate axis and asymptotes.	Students should recognize the form of the equation and hence the orientation of the graph. The parameters a, b and c can then be determined. It should be noted that, unlike the ellipse, <u>a</u> is not always greater than <u>b</u>.		P.393	P.412-418
19. Derive the relation between the parameters a, b and c for the hyperbola. $(a^2 + b^2 = c^2)$			P.392	P.412-418
20. Determine an equation and sketch the hyperbola given: a. transverse axis and conjugate axis b. foci and length of one axis c. equation of an asymptote and a point on the hyperbola	A rough sketch can help determine the required parameters. In some cases, substitution of coordinates of a point may be required to find one of the parameters. Hyperbolas with centres off the origin are not included.		P.393-395	P.418
21. Solve applied problems related to the hyperbola.			P.395	P.419-424

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
C. SEQUENCES, SERIES, LIMITS				
1. Recognize the differences between: a. a sequence and a series b. finite and infinite sequences			P.434-438	a.P.254 & 266 b.P.254-257 P.266-278
2. a. Recognize and define arithmetic sequences and series and state the common difference (d). b. Derive and apply: 1. the general term formula $a_n = a_1 + (n - 1) d$ 2. the sum formula $s_n = \frac{n}{2} (a_1 + a_n)$ c. Apply formulas to problems involving arithmetic sequences and series.	The formula $S_n = \left(2a,+(n-1)d\right)$ may be more convenient for some problems and should be derived from $S_n = \frac{n}{2} (a_1 + a_n)$	Ben Business, a bright young college graduate, sells popsicles for 14¢ each, while only having to buy them from the wholesaler for 3½¢ a piece. As the cooler weather approaches, Ben finds himself reducing the price of popsicles by one cent a week, while his distributor, trying to maintain his profit, raises the wholesale price by ½¢ a week. If Ben is really smart when should he pull out? (7 weeks) Quasi Motors Inc. produces 12 units of farm machinery a day and sells them the same day to retail distribution centres across the country. If production costs are rising at $3 per unit per day, the competitors are forcing the sales price down at $5 per unit per day, when should Quasi Motors stop production if production costs are now $10 and the price of machinery is $150 per unit? How much profit will they make in that time? ($13,872)	P.439-445 P.460-465	a.P.257-261 b.P.257-261 P.268-272 c.P.257-261 P.268-272
3. a. Recognize and define geometric sequences and series and state the common ratio (r).			P.446	a.P.262-266

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
3. Continued b. Derive and apply: i. the general term formula $a_n = a_1 r^{n-1}$ ii. the sum formulas $S_n = \frac{a_1 (r^n - 1)}{r - 1} \quad r \neq 1$ $S_n = \frac{ra_n - a_1}{r - 1}, \quad r \neq 1$		When changing car engines, a mechanic often uses a pulley system. If he and his assistant can easily manage to pull with 120 N of force, how many pulleys will they need to lift a car engine resting in the car with 3,000 N of force? (5.6 rounded to 6) If 100 is invested at 9% compounded annually, show how the amount grows over a term of n years. 	P.447-452 P.453-456 P.466-474	b.P.262-266 P.272-274
c. Apply formulas to problems involving geometric sequences, and series, with special emphasis being given to the mathematics of finance: i. Difference between and applications of simple and compound interest. ii. Use tables to determine accumulated and present value accounts involving compound interest over different time periods. iii. Illustrate the various annuities by using line diagrams.		Notice that the amounts at the ends of successive years form the terms of a geometric sequence where: $a = 100\ (1.09)$ $r = 1.09$ and a_n is the nth term of the sequence. The amount after n years: $a_n = ar^{n-1}$ $a_n = 100\ (1.09)\ (1.09)^{n-1}$ $= 100\ (1.09)^n$		i.P.274-277 ii.P.278-280 iii.P.274-280

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
3. c. Continued iv. Apply geometric series to both accumulated and present value annuities with both identical and differing interest and payment periods.	Apply concepts of annuities to relevant amortization plans, house mortgages, auto financing and interest-accumulations.	An insurance policy pays $30,000 at age 60 which may be taken as a lump sum or in 30 equal half-yearly payments with the principal interest at 8% compounded semi-annually. If the first payment is made 6 months after the 60th birthday, how large is each payment?		iv. P.280-287 P.288-289
4. Generate the terms of a series using sigma notation (Σ).			P.457-459	P.266-268
5. Determine the limits of various functions.	Functions involved should be limited to those which after appropriate algebraic simplification can be analyzed using the fact that $\lim_{n \to \infty} \frac{1}{n} = 0$		P.479-480	P.453-456
6. Recognize the differences between infinite convergent and divergent sequences.			P.477-478	P.446-453
7. Find the limits of infinite convergent sequences.			P.475-476	P.442-445
8. Find the sums of infinite convergent series.			P.477-478	P.446-451
9. Solve problems involving infinite geometric series.		Half-life of radioactive materials.	P.479	P.451-453

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
D. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS				
1. Maintain previous skills: a. Frequency distribution b. Measures of central tendency c. Measures of dispersion			P.281-299	a.P.367-373 b.P.336-337 c.P.374-375
2. Illustrate and develop the normal distribution	1. Comparing the normal curve to histograms of progressively narrower intervals (or frequency polygons) shows how smooth curve is obtained from actual data. 2. Relate measures of central tendency and dispersion to the normal curve.	Predictions, life expectancy, mortality tables.	P.306-310	P.376-379
3. Develop and apply standard deviation.	Mean deviation is a good way of introducing standard deviation. Standard measure $z = \frac{x - x}{\sigma}$ may be introduced as a measure independent of units used. This allows easy use of tables involving areas under the normal curve.	Quality control in industry.	P.300-305	P.374-379
4. Introduce probability using an experimental approach.	Introduction to such terms as outcome, sample space and event would be appropriate.		P.310-316	P.356-360
5. Apply probability to theoretical frequency distribution.	Relate probability to previous objectives as a theoretical long term frequency. Estimated probability is the relative frequency of occurrence of the event when the number is very large.	Find the probability that in 120 tosses of a fair coin between 40% and 60% will be heads.  $\bar{x} = 60$	P.316-324	P.383-388

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES MATH IS	FMT
5. Continued		Given $\sigma = 5.5$ 40% of 120 = 48 60% of 120 = 72 48 converted to standard units $= \frac{48 - 60}{5.5} = -2.2$ 72 converted to standard units $= \frac{72 - 60}{5.5} = 2.2$ Required probability = area under the normal curve between $z = -2.2$ and $z = 2.2$ $P = 2(.4861) = .9722$		

OBJECTIVES	COMMENTS	APPLICATION	PRESCRIBED REFERENCES MATH IS	FMT
E. LOGARITHMS				
1. Maintain previous skills on exponents.			P.VII P143-P155	P.135-148
2. Identify and graph exponential functions.	This study of logarithms should emphasize the functions, their graphs and their use in applications.	Logarithms are frequently used to describe rates of growth, rates of decay, sound intensity, light intensity and earthquake intensity.	P.156-160	P.148-152 P.153-157
3. Convert equations from exponential form to logarithmic form and vice versa.		Sample Questions	P.178-179	P.159-161
4. Solve logarithmic equations by converting to exponential form.	$\log_{16}x = -\frac{5}{4}$ $\log_x \frac{27}{8} = -3$ $\log_2 \frac{1}{16} = x$	a) Loudness of sound is measured on a decibel scale according to the formula: D - 10 Log (L), where D is the number of decibels of sound and L is the loudness or intensity of the sound.	P.179-180	
5. Define the inverse of an exponential function in logarithmic form.	Emphasize the relationship between exponential and logarithmic functions.	How many times louder is sound of 52 decibels than sound of 37 decibels?	P.177-180	P.158-161
6. Evaluate expressions and solve equations involving logarithmic form and exponential form.		b) A scientist predicts that 20 grams of a radioactive substance will decay in such a way that after (T) days the number of grams remaining (G) may be estimated according to the formula	P.184-185 P.200-202	P.161-164
7. State and use the basic laws or properties of logarithms, products, quotients, powers and roots.	Limit the time spent using logarithms to calculate products, quotients, roots or powers. Relate these to corresponding properties of exponents.	$G = 20X^{-T/15}$ Determine to one decimal place, the number of days required for the substance to reduce to 8 g.	P.182-189 P.193-198	P.161-164

OBJECTIVES	COMMENTS	APPLICATION	PRESCRIBED REFERENCES MATH IS	PRESCRIBED REFERENCES FMT
8. Use logarithms to solve practical problems.	Students should be able to recognize that $\log (2a) = \log 2 + \log A$ $\log \frac{4}{a} = \log 4 - \log a$ $\text{Log } a^3 = 3 \log a$ and vice versa	c) The length of time to double your money at 12% is found by solving $2 = (1.12)^n$ for n.	P.160-161 P.165-174 P.181-182 P.189-190 P.199-202	P.165-172

MATHEMATICS 33:
PRIMARY REFERENCE CORRELATION

MATHEMATICS 33: PRIMARY REFERENCE CORRELATION

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
A. RELATIONS AND FUNCTIONS			
1. Define a relation and its domain and range, utilizing graphs and algebraic statements.	Relations may be defined by ordered pairs, graphs or open sentences. Family relations actually are examples which can be used to illustrate mathematics. Emphasis should be graphical.		
2. Define the term function.	A function may be defined as "a relation which has no 'fickle pickers'".	Experimental activities help motivate the students to learn this concept. For example: $R = \{(2,3)\ (2,4)\ (3,5)\}$ may be represented as: $2 \rightarrow 3$, $2 \rightarrow 4$, $3 \rightarrow 5$ using the matching notation, 2 is a 'fickle picker' because it picks more than one member of the range. $S = \{(2,3)\ (4,5)\ (6,7)\}$ may be represented as: $2 \rightarrow 3$, $4 \rightarrow 5$, $6 \rightarrow 7$ there are no 'fickle pickers', so this relation is a function	
3. Use functional notation f(x) for particular values of x.			P.104
4. Define and graph a linear function.			P. 86
5. Show the relationship between the graphs of linear and quadratic functions and the roots of the corresponding equations.			P. 87 72 73

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
B. TRIGONOMETRY			
1. Maintain previously developed skills.	Review terms associated with rectangular coordinate system; review the theorem of Pythagoras. Find coordinates of points on the x-axis and y-axis. Note: The relationship between degrees, minutes and decimal degrees should be discussed.		P.172-191 297 210-211
2.			
3.			
4.			
5.			
6.			
7. Solve simple trigonometric equations.	Coverage should include equations of the linear and quadratic format. For example, $3 \cos \theta + 1 = 2$, $2 \sin^2\theta + \sin \theta = 1$		P.233-238
8.			

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
9. Derive and apply the following identities: a. Quotient relations: $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$ b. Reciprocal relations: $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$ c. Pythagorean relations: $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$			P.235-238
10.			
11.			
12.			
13.			
14.			
15.			

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
16. Determine the value of a trigonometric ratio of any angle.	Angles should be restricted to positive angles less than or equal to 360 degrees Related angles may be illustrated by geometric ideas such as congruent triangles or reflections. In actual practice students will find it easier to use the 3-step reduction of angles method. 1. Find the related angle. 2. Use the same function as the given one. 3. Determine the sign of the function.		P.216-224
17. Solve oblique triangles by using the sine law and/or cosine law.	Law of Sines: Case II - Problems in which two sides and an angle opposite one of them are given may have more than one solution depending on the relative measure of the sides (ambiguous case). Better students may wish to pursue an analysis of this case in detail.		P.192-197 207-208
18. Apply the sine law and cosine law to practical problems.		In surveying, finding inaccessible distances. In aerial navigation, determine which course to fly to counteract the effects of the wind.	P.198-199 208-209
19.			

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
C. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS			
1. Maintain previous skills. a. Frequency distribution b. Measures of central tendency c. Measures of dispersion			P.25-40
2. Illustrate and develop the normal distribution.	1. Comparing the normal curve to histograms of progressively narrower intervals (or frequency polygons) shows how the smooth curve is obtained from actual data. 2. Relate measures of central tendency and dispersion to the normal curve.	Predictions, Life expectancy, Mortality tables.	P.42-44
3. Develop and apply standard deviation.	Mean deviation is a good way of introducing standard deviation.	Quality control in industry.	P.41-44
4. Introduce probability using an experimental approach.	Introduction to such terms as outcomes, sample space and event would be appropriate.		P.45-54

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
D. QUADRATIC FUNCTIONS, EQUATIONS AND APPLICATIONS			
1. Identify and express quadratic functions in the form $y = ax^2 + bx + c$ where $a, b, c \in R$, $a \neq 0$.			P.86-87
2. Identify and express quadratic equations in the form $ax^2 + bx + c = 0$ where $a, b, c \in R$, $a \neq 0$.			P.72
3. Graph a quadratic function using a table of values.	Students should become familiar with the graphs of quadratic functions of the forms: $y = ax^2$, $y = ax^2 + bx$, and $y = ax^2 + bx + c$.		P.88
4. Find the vertex, axis of symmetry, domain range and maximum or minimum value of a quadratic function from its graph.	The importance of the vertex and axis of symmetry can be illustrated using appropriate problem solving applications.	A road passes under a railroad overpass in the form of a parabolic arch 5 metres high (at the apex) and 20 metres wide with equation $y = 5 - \frac{x^2}{20}$ Find the height of the tallest truck that can pass under the arch. Assume the truck is 3 metres wide, that it stays on its own side of the road, and that the centerline of the road passes directly under the apex of the arch.	P.88
5. Use the formula for vertex and axis of symmetry if the quadratic function is given in the form $y = ax^2 + bx + c$.	It is suggested that the formula be derived before it is used.		

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
6. State the relationship between the graph of a quadratic function and the roots of the corresponding equation.			P.92-94
7. Write quadratic equations in the form $ax^2 + bx + c = 0$ and specify the value of a, b, c.	Initial graphing of quadratics by a table of values acquaints students with the more important points of a quadratic and also develops an appreciation for graphing quadratics using these critical points.		P.94-95
8. Use the method of completing the square of a quadratic function to find the vertex, axis of symmetry, range and maximum or minimum value. Draw the graph using this information.	The value of (a) should be restricted to the set of integers.		P. 92-94
9. Solve problems involving the maximum or minimum value of a quadratic function.		a. The Wishbones have 30 metres of fence with which to make a rectangular dog run. If they use a side of the house as one side of the run, what dimensions will give the maximum area? b. Fast Freddie's Theatre charges $4.00 per ticket, and it has had a full house of 400 nightly. The manager estimates that the ticket sales would decrease by 50 for each $1.00 increase in the ticket cost. What is the most profitable price to charge? (A simple arithmetic solution using a table of values could be used to introduce the concept of maximum value). e.g.: N: 400, 350, 300, 250 U: 4, 5, 6, 7 I: 1600, 1750, 1800, 1750	P.96-100

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
9. Continued.		c. The collision impact (I) of an automobile with mass (m) and speed (s) is given by the formula $I = kms^2$? If the speed is tripled, what happens to the collision impact of a 1000 kg car?	
10. Compute the real roots of a quadratic equation by: a. factoring b. using the quadratic formula or completing the square.			P.74-77
11. Define and evaluate the discriminant of a quadratic equation.			P.78-79
12. State the nature of the roots by examining the discriminant.			P.79
13. Solve problems whose solutions are based on quadratic equations.		a. A club bought a snowmobile for $720.00, planning to divide the expenses equally among the members. However, two members withdrew from the club, increasing each share by $4.00. How many members were in the club originally? b. How wide a strip must be sodded around a square court 10 metres on a side, so that one-half of the court is sodded?	P.79-84

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
13. Continued.		c. A 6 centimetre square is cut from each corner of a square piece of sheet metal. The sides are folded up to form an open box having a volume of 600 cc. What is the length of a side of the original square piece of sheet metal? d. Two students agree to share the work of cutting the grass on a large lot measuring 80 metres by 60 metres. One student starts by cutting a strip all of the way around the lot and continuing around in this way. How wide will this trip be when half the work is completed?	

OBJECTIVES	COMMENTS	APPLICATION	PRESCRIBED REFERENCES AMT
E. LOGARITHMS			
1. Maintain previous skills on exponents.			P.159-160
2. Identify and graph exponential functions.	This study of logarithms should emphasize the functions, their graphs and their use in applications.	Logarithms are frequently used to describe rates of growth, rates of decay sound intensity, light intensity and earthquake intensity. <u>Sample Questions</u> a) Loudness of sound is measured on a decibel scale according to the formula: D - 10 Log (L), where D is the number of decibels of sound and L is the loudness or intensity of the sound. How many times louder is sound of 52 decibels than sound of 37 decibels? b) A scientist predicts that 20 grams of a radioactive substance will decay in such a way that after (T) days the number of grams remaining (G) may be estimated according to the formula $G = 20X^{-T/15}$ Determine to one decimal place, the number of days required for the substance to reduce to 8 g.	P.148-151
3. Convert equations from exponential form to logarithmic form and vice versa.			P.161-164
4. Solve logarithmic equations by converting to exponential form.	$\log_{16}x = -\frac{5}{4}$ $\log_x \frac{27}{8} = -3$ $\log_2 \frac{1}{16} = x$		
5. Define the inverse of an exponential function in logarithmic form.	Emphasize the relationship between exponential and logarithmic functions.		
6. Evaluate expressions and solve equations involving logarithmic form and exponential form.			P.164-170
7. State and use the basic laws or properties of logarithms, products, quotients, powers and roots.	Limit the time spent using logarithms to calculate products, quotients, roots or powers. Relate these to corresponding properties of exponents.		

OBJECTIVES	COMMENTS	APPLICATION	PRESCRIBED REFERENCES AMT
8. Use logarithms to solve practical problems.	Students should be able to recognize that $\log (2a) = \log 2 + \log A$ $\log \frac{4}{a} = \log 4 - \log a$ $\log a^3 = 3 \log a$ and vice versa	c) The length of time to double your money at 12% is found by solving $2 = (1.12)^n$ for n.	

MATHEMATICS ELECTIVES

ABSOLUTE VALUE

RECOMMENDATIONS: MATH 20, 30, 23, 33.

In some problems only the magnitude of a value is required. E.g., How much did the temperature increase from 9:00 a.m. to 3:00 p.m. if it were - 8°C at 9:00 a.m. and 4°C at 3:00 p.m.?

We would disregard the direction on a number line in order to solve the above. In other words, we would use the absolute value of the numbers.

References:

1. Ebos, F., and Tuck, B., *Math Is 4,* Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: Intermediate,* Toronto: McGraw-Hill Ryerson Ltd., 1978.

3. Carter, J., Clark, J., Porter, B., and Stouffer, M., *Mathematics Alive 3,* Toronto: Copp Clark Publishing, 1978.

4. Bye, M., and Elliott, H., *Math Probe 3,* Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use the absolute value symbol correctly.	
2. Use absolute value to indicate the measure of the distance between the point and the origin on a number line.	
3. Use absolute value to tell the distance between any two points on the number line.	
4. Solve equations |like $2x\| - 4 = 8$	
5. Find the solution sets of inequalities like $\|3x\| < 12$ and graph the solution set.	

AREA AND VOLUME

RECOMMENDATIONS: MATH 20, 30, 23, 33

Observation tells us that geometric figures are common in everyday life. It is often necessary to find areas or volumes in order to solve everyday problems.

References:

1. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Applied Mathematics for Today: Introduction*, Toronto: McGraw-Hill Ryerson, Ltd., 1980.

2. Ebos, F., and Tuck, B., *Math is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1970

OBJECTIVES	COMMENTS/ACTIVITIES
1. Determine the area (using the pertinent formula) for each of the following plane figures: triangles, squares, rectangles, parallelograms, trapezoids and circles.	Using different floor plans have students find the area of floors so that wall-to-wall carpet may be laid.
2. Determine the total area (or area of different regions) of various polyhedrons (prisms and pyramids), cylinders and cones.	Wall papering is popular. Have students find the quantity of wallpaper needed to cover walls in rooms of various shapes. Wallpaper (or adhesive covering material) is often used to decorate various household items like wastepaper baskets. Have students suggest different items and calculate the quantity of covering required.
3. Determine the volume of various polyhedrons (prisms and pyramids), cylinders, cones and spheres.	Using structural forms with various dimensions students could calculate the number of M^3 of concrete to be used. The capacity of swimming pools, silos, etc., of varying shapes and dimensions could be calculated.

NOTE: A review of (a) The theorem of Pythagoras and (b) π might be required.

ARRANGEMENTS AND SELECTIONS

RECOMMENDATIONS:

MATH 30, 33

References:

1. Nichols, E., Heimer, R., and Garland, E., *Modern Intermediate Algebra*, Toronto: Holt, Rinehart and Winston of Canada.

2. Johnson, R., Lendsey, L., Slesnick, W., and Bates, G., *Algebra and Trigonometry*, Don Mills, Ontario: Addison-Wesley Publishing Company.

3. Elliott, H., Fryer, K., and Gardner, J., *Algebraic Structures and Probability*, Toronto: Holt, Rinehart and Winston of Canada.

4. Travers, Dalton et al, *Using Advanced Algebra*, Toronto: Doubleday Canada Ltd.

5. Dolciani, M., Berman, S., and Wooton, W., *Modern Algebra and Trigonometry*, Don Mills: Thomas Nelson and Sons (Canada) Ltd.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Evaluate and/or simplify expressions involving factorials and/or ${}_nP_r$.	
2. Solve word problems involving linear arrangements of n different objects.	
3. Solve word problems involving a linear arrangement of r objects, given n different objects.	
4. Solve word problems involving linear arrangements of one or more objects at a time, given n different objects.	

OBJECTIVES	COMMENTS/ACTIVITIES
5. Solve word problems involving linear arrangements of n objects, if some of the objects are the same.	
6. Solve word problems involving a restricted linear arrangement of objects.	
7. Solve equations involving the application of the formula $_nP_r = \frac{n!}{(n-r)!}$	
8. Solve word problems involving the application of circular arrangements of n different objects.	
9. Solve word problems involving circular arrangements of n different objects, with restrictions.	
10. Solve word problems involving circular arrangements on a ring.	
11. Evaluate and/or simplify expressions involving factorials and/or $_nC_r$.	
12. Solve word problems involving a selection of r objects, from n different objects.	
13. Solve word problems involving selections of one or more objects at a time, from n different objects.	

OBJECTIVES	COMMENTS/ACTIVITIES
14. Solve word problems involving a restricted selection of objects.	
15. Solve equations involving the application of the formula $_nC_r = \frac{n!}{r!\,(n-r)!}$	

COMPLEX NUMBERS

RECOMMENDATIONS: MATH 20, 30, 33.

Complex numbers are a natural outgrowth of the study of quadratic equations. Development of a new number system through the introduction of a new symbol can be illustrated. For example, in expanding the system No to I, we merely introduce a new symbol: the minus sign. Similarly for the complex numbers, we introduce a new symbol, i, to represent $\sqrt{-1}$. This allows us to solve quadratic equations which have no solution in the real number system.

Although few practical applications of complex numbers exist at a level which high school students can understand, complex members are used extensively by electrical engineers. Several examples of this, at a very basic level, are included in reference 3.

References:

1. *Algebra Two and Trigonometry*, Vogeli et al, Silver-Burdett Company.
2. *Holt Algebra 2 With Trigonometry*, Nichols et al, Holt, Rinehart and Winston, 1974.
3. *Using Advanced Algebra*, Travers et al, Laidlaw Publishers, 1975.
4. *Algebra and Trigonometry: Structure and Method (Book 2)*, Dolciani et al, Houghton, Mifflin, 1977.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Demonstrate the need for a new number system beyond the real numbers in order to solve equations of the form $x^2 + 1 = 0$.	
2. (a) Perform simple arithmetic operations with complex numbers of the form $a = bi$. (b) Evaluate the conjugate and use it in division of complex numbers. (c) Solve for unknowns for equations in the form $2 + 3i = x + yi$.	

OBJECTIVES	COMMENTS/ACTIVITIES
3. Graph complex numbers on an Argand diagram, (rectangular, coordinate system) using a vector to represent the complex number.	
4. Evaluate the absolute value (modules) of a complex number and interpret it as the length of a vector on an Argand diagram.	
5. Solve quadratic equations which have non-real roots.	
6. Solve higher degree equations by special factoring methods (sum or differences of cubes, differences of squares).	
7. Find roots of a number algebraically. (e.g., $x^3 + 1 = 0$ can be used to find the 3 cube roots of -1).	
8. Find the roots of a number of graphical means, using an Argand diagram. (All roots represented by vectors with an equal length).	
9. Solve application problems of complex numbers (impedence of an electrical circuit).	

COMPUTER LITERACY

RECOMMENDATIONS: MATH 20, 30, 23, 33

Many of today's students are becoming acquainted with the workings of the computer and the many uses to which it can be put. An elective component of computer literacy should provide the student with an introduction to the field of computers with a provision for students who are interested to continue work in computer science.

References:

1. *The Mathematics Teacher*, National Council of Teachers of Mathematics, February, 1980.

2. Brown, J. R., *Instant Basics*. Dilitherim Press, Forest Grove, Oregon, 1978.

3. Dwyer, T. and Critchfield M., *Basis and the Personal Computer*, Addison-Wesley, Don Mills, Ontario, 1978.

4. Lien, David, *Radio Shack Computer Language*. Fort Worth, Texas: Tandy Corporation, 1978.

At present there are a variety of computer companies with many changes occuring rapidly. It would be advisable to become acquainted with the variety of hardware and software available on the market.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Develop an appreciation of the role of computers in our society.	Give examples of how computers are used in data processing, mathematics, science and other fields. Describe the general categories and types of computers in society. Summarize the historical events and more recent developments that have resulted in present-day computers. Describe the duties and education needed for various types of computer-related occupations. Give examples of how computers have been misused in society.

OBJECTIVES	COMMENTS/ACTIVITIES
	Define concepts like artificial intelligence, dehumanization, miniaturization.
2. Obtain basic understanding of the computer as a machine: its capabilities and limitations.	Identify the essential components of a modern computer. Identify the function and relation of each component to the whole computing system. State whether or not a given problem is suitable for solution on a computer. Give examples of problems or situations that cannot be solved by a computer.
3. Develop the algorithmic process of thought.	Solve standard types of mathematical and logical problems by using various techniques and strategies. Give an example of an algorithm. Flowchart a simple algorithm.
4. Acquire a working knowledge of a programming language.	Differentiate between machine language, a programming language and job control language. Prepare and run a simple computer program using the fundamental features of the language BASIC. Program in BASIC using advanced features like looping, subscripted variables, character strings and library functions.
5. Develop skills enabling the student to use the computer for problem solving.	Solve a given mathematics or data processing problem by using a computer. Describe the solution to a problem by providing the appropriate documentation, flowcharts, program listing and sample runs.

CONSUMER MATHEMATICS

RECOMMENDATIONS: MATH 20, 30, 23, 33.

It is becoming more and more important that students have some understanding of how mathematics is related to the consumer. A working knowledge of how the basic skills in mathematics are applied to consumer topics such as accessing money, spending money and the management of money is essential for a student to become a good citizen.

References:

1. *Mathematics in Life*, Gage Publishing Co.
2. *Mathematics Plus*, Houghton Mifflin
3. *Mathematics for the Real World*, Merrill/Bell and Howell Publishing Co.
4. *Mathematics for Daily Use*, Doubleday Publishing Co.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To develop an appreciation of how money is obtained. (a) Jobs - Descriptions - Applications - Wages	Identify different careers and discuss the wages paid for the related work. Identify what is needed for the accomplishment of different jobs. Discuss filling out applications and interviews.

OBJECTIVES	COMMENTS/ACTIVITIES
1. (Continued)	
(b) Payroll - Methods of payment - Deductions · Tax · Pension · Benefits	Have students discuss and analyze real or imaginary paychecks.
(c) Selling Products	Have students participate in such activities as a school store, concession or canteen.
(d) Record Keeping.	Discuss and practice the necessity for good record keeping.
2. To develop an understanding of wise and thrifty spending habits.	
(a) Buying	
- Comparative buying	Students can check out the cost of a shopping list of some everyday needs. Prices can be compared from store to store, by brand and by size. Prices could be prepared over a period of time to determine price changes and trends.
(b) Credit Buying - Types of credit - Types of payment - Interest rates - Installment buying - Sources of borrowed money	Students could determine the cost of buying such items as a car.
(c) Operation Costs	Students could determine the cost of operating a car for a period of a year. Things to take into account are: purchase price, licence, insurance gasoline, service and maintenance, depreciation.

OBJECTIVES	COMMENTS/ACTIVITIES
2. (continued)	
(d) Bank Accounts	Students can study the various bank accounts probably best by an organized field trip to a local bank.
(e) Food, Shelter and Clothing	Generate activities with the students to have them role-play situations about the basic necessities of life.
3. Mathematics of Money Management	
(a) Insurance - Life - Property - Automobile	Have students study the various types of insurance programs in order to determine the advantages and disadvantages of each type.
(b) Home Ownership and Renting	Have students discuss the sale and rental of homes with the local real estate firms.
(c) Taxation	Make a study of income tax and fill out an income tax form. Discuss the local tax structures within a community.
(d) Investments	Have students make a study of the stock market.

HISTORY OF MATHEMATICS

RECOMMENDATIONS: MATH 20, 30, 23, 33

The history of mathematics can provide many interesting discussions and lends itself to many interesting projects for the classroom. A look at historical topics is unlimited and is dependent only upon the creativity of the teacher.

Reference:

1. *Historical Topics for the Mathematics Classroom*, Thirty-first yearbook, National Council of Teachers of Mathematics, 1969.

   NOTE: This yearbook also contains an excellent bibliography of the many books and articles available on historical topics in Mathematics.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To acquire an appreciation for the historical development of mathematics from counting to modern day computers.	Discuss various types of early developments in mathematics. Many of these can be set in activity stations, as display areas or in written project form.
2. To humanize mathematics by looking at the lives of some of the mathematicians.	Investigate the lives of the famous mathematicians by using different methods such as: reporting, writing essays, role-playing, discussions, films.
3. To interrelate mathematics with other subject areas such as science, music, social studies and art.	
4. To familiarize students with the use of various mathematical instruments which have been developed throughout the years.	Discuss, demonstrate the use of, or construct the basic mathematical instruments for:

OBJECTIVES	COMMENTS/ACTIVITIES
4. (Continued)	(a) Measurement - sundial - waterclock - transit - sextant - angle mirror - micrometer - caliper - trundle wheel (b) Calculations - slide rule - logarithms - calculators - Napier's bones - abacus - computers

INEQUALITIES

RECOMMENDATIONS: MATH 20, 30, 23, 33

All or most of our past experience in mathematics has dealt with equalities. We can make use of this experience to solve inequalities since inequalities are solved in the same manner as equalities except when multiplying or dividing by a negative. In this case the direction of the inequality sign must be changed.

References:

1. Burns, A., Pinkney, R., and Del Grande, J. *Mathematics for a Modern World: Book 3;* Second Edition, Toronto, Gage Educational Publishing Ltd., 1976.

2. Ebos, F., and Tuck, B., *Math is 4,* Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Solve inequalities.	
2. Graph the solution sets of inequalities.	

LINEAR PROGRAMMING

RECOMMENDATIONS: MATH 20, 30, 33

Linear programming involves the attempt to maximize results and minimize efforts to produce those results. Such attempts originated during World War II when the Allies attempted to maximize production and minimize costs. Manufacturing problems involving numbers of items and various other constraints, such as time required for production, can be analyzed in a systematic way using linear equations and inequalities.

The fundamental assumption of linear programming is: the maximum value of the parameter P, for the relation $P = Ax + By$, occurs at one of the vertices of the polygonal region determined by the various constraints in the problem. The maximum value of P can be found by examining these vertices.

References:

1. Ebos and Tuck, *Math is 4*, Thomas Nelson and Sons, 1979.
2. Travers, Dalton et al, *Using Advanced Algebra*, Doubleday, 1977.
3. Wigle, Jenning and Dowling, *Mathematical Pursuits Three*, Macmillan of Canada, 1977.
4. Dotton, Knill and Seymour, *Applied Mathematics for Today*, McGraw-Hill Ryerson, 1976.
5. Hanwell, Bye and Griffiths, *Holt Mathematics 4 (Second Edition)*, Holt, Rinehart and Winston, 1980.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Define the following terms: parameter, region, constraint, maximum point.	
2. Draw the graph of a linear equation in the form $Ax + By = C$.	
3. Draw the graph of a region defined by an inequality.	

OBJECTIVES	COMMENTS/ACTIVITIES
4. Determine a region bounded by several inequalities.	
5. Find the intersection of 2 linear equations by graphical or algebraic means.	
6. Find the maximum value of a perimeter P defined by $P = Ax + By$ for a given set of constraints.	
7. Determine the value of a parameter P at each of the vertices of the polygon which results from graphing all the constraint conditions on x and y.	
8. Apply linear programming to solving business-oriented problems.	

MATH ART

RECOMMENDATIONS: MATH 20, 30, 23, 33

This unit is intended to give students an opportunity to apply in an interesting way some of the skills and concepts learned in Geometry (both Euclidean and Analytic). Once a student learns some of the simple principles of the artwork, he can create his own designs.

References.

1. *Art 'n Math,* Billings, Campbell and Schwandt (Action Math Associates, Inc.).

2. *Creating Escher - Type Drawings,* Rannucci and Teeters (Creative Publications).

3. *Graph Gallery,* Boyle, (Creative Publications).

4. *Paper Folding in the Classroom,* Johnson (NCTM Publications).

5. *Creative Constructions,* Seymour (Creative Publications)

6. *Line Designs,* Seymour (Creative Publications).

7. *Mathematics Teacher (NCTM)*

   (a) Tangram Mathematics, February, 1977, pp. 143-146
   (b) The Artist as Mathematician, April, 1977, pp. 298-308
   (c) Filing (tesselations), March, 1978, pp. 199-202.
   (d) Transformation Geometry and the Artwork of M.C. Escher, December, 1976, pp. 647-652.
   (e) Creativity With Colors, March, 1976, pp. 215-218.

8. *Geometry,* Jacobs, (W. H. Freeman Publishing).

OBJECTIVES	COMMENTS/ACTIVITIES
1. Graphing pictures (a) Given a set or ordered pairs, locate these on a coordinate system and join them to form a picture.	

OBJECTIVES	COMMENTS/ACTIVITIES
1. Graphing Pictures (continued)	
(b) Given a set of linear equations, each with a restricted domain, draw the lines on a coordinate system to form a picture.	Student writes his name or suitable message in block letters on graph paper, then analyzes each letter to write the equations which represent them (restrict to straight lines).
2. Line Designs	
(a) Draw line designs within one angle.	Student creates his own pattern using basic construction techniques for angles. Alternate rectangles can be colored to form a pattern. The same approach can be used with colored thread and small nails on a sheet of plywood to create string designs.
3. Geometric Constructions	
(a) Perform basic constructions related to line segments, angles, perpendicular and parallel lines.	Student analyzes and recreates simple drawings from patterns.
(b) Draw a polygon within a circle using ruler and compass (e.g., hexagon, polygon, square).	Student creates and colors his own designs.
4. Tesselations	
(a) Draw a tesselation based on a rectangle, a triangle or other polygons.	Students find examples of tesselations; e.g., linoleum, floor tiles, wallpaper, brick wall, etc.
5. Escher Drawings	
(a) (An extension of tesselations). Draw a simple tesselation based on Escher's work and analyze how it is drawn.	Students create their own Escher - type drawings, Students analyze some of Escher's paintings to find geometric contradictions (e.g., "Belvoir").

OBJECTIVES	COMMENTS/ACTIVITIES
6. Paper Folding	
(a) Demonstrate the following concepts using paper folding: - angle bisector - bisection of a segment - line perpendicular to a given line - centre of a circle	
(b) Construct a parabola, hyperbola and ellipse using paper folding.	
7. Tangrams	
(a) Rearrange polygonal shapes to form geometric figures.	Using the 7 pieces of a tangram square, form rectangles, isosceles triangles, parallelograms and trapezoids.
(b) Illustrate basic properties of polygons (e.g., form an isosceles trapezoid and show the non-parallel sides are equal by moving the component parts around).	Use the tangram pieces to make up figures such as the letter "T", the number "1", a house, a cat, etc.

MATHEMATICAL INDUCTION

The name, mathematical induction, is somewhat misleading because the process is deductive in nature, leading to a firm conclusion. It is usually employed in proving the validity of a statement involving all positive integral values of "n".

For example, one method of finding a square root on an ordinary computing or adding machine is based on the fact that the sum of "n" odd integers is equal to n^2.

$$1 = 1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

$$1 + 3 + 5 + \text{-----} + (2n-1) = n^2$$

If this can be proven for (a) specific values of n and, (b) when nth position = k as well as k + 1 where right side equals left side then the conclusion (c) is verified.

References:

1. Johnson, R.E., Lendsey, L.L., Slesnich, W.E., Bates, G.E., *Algebra and Trigonometry*, London: Addison-Wesley Publishing Co., 1976.

2. Del Grande, J.J., Duff, G.F.D., Egsgard. J.C., *Mathematics 12 - Third Edition*, Toronto: Gage Publishing Ltd., 1980.

3. Vance, E.P., *Mathematical Induction and Conic Sections*, London: Addison-Wesley Publishing Co., 1971.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use mathematical induction to prove various additive algorithms.	

OBJECTIVES	COMMENTS/ACTIVITIES
2. Use mathematical induction to prove various division algorithms.	
3. Use mathematical induction to prove trigonometric algorithms.	

MATHEMATICS FOR INDUSTRY

RECOMMENDATIONS: MATH 20, 30, 23, 33

Many practical applications for mathematics are available through the Industrial Arts department. Consultation with the Industrial Arts teachers can provide many interesting projects for students.

For example, the Math Modules produced by General Publishing Company provide many examples in the areas of drafting, machine shops and consumer mathematics.

Preparation of units, projects and activities in this section are completely dependent upon the creativity of the teacher.

MATHEMATICS OF ACCOUNTING

RECOMMENDATIONS: MATH 20, 30, 23, 33

The value of accounting skills in the everyday life of all people has increased dramatically in recent years. As a result all students could benefit from a basic knowledge of how mathematics is applied to the field of accounting. The treatment of basic accounting procedures could be designed to permit interested students to further pursue accounting courses in the Business area.

References:

The Business Education department of the school should have many practical references to use. Consultation with the Business Education teachers would be desirable in setting up an elective in this area.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Emphasize the importance of maintaining adequate accounting records in business and personal life.	
2. Emphasize the importance of basic mathematics skills to the field of accounting.	
3. Improve ability in problem-solving and logical thinking through real-life applications.	
4. Develop traits of neatness, accuracy and the ability to interpret and analyze basic accounting records.	

MATRICES

RECOMMENDATIONS: MATH 20, 30, 33

Tables are very useful ways of arranging information. Data from tables can be set in a more concise form called a matrix where we write this data in columns and rows.

Reference:

1. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To recognize and use the matrix as a concise form of arranging information.	Construct a 3 x 3 magic square. Then rearrange the numbers in the cells so that the sum of each new row, column and diagonal is also equal. How many arrangements are there if the square is to remain a magic square? Do likewise for a 4 x 4 and a 5 x 5 magic square.
2. To comprehend the difference between columns and rows.	Using everyday situations construct matrices with different orders. E.g., team records.
3. To multiply matrices.	
4. To apply matrices in solving problems.	

POLYNOMIAL FUNCTIONS

The study of polynomial functions is an opportunity to extend the previously studied topics of polynomial functions, linear and quadratic. Such algebraic processes as factoring and division become steps in a process to determine the roots of polynomial equations of second degree or higher. Sketch the graphs of polynomial functions.

References:

1. Nichols, El, Heimer, R., and Garland, E., *Modern Intermediate Algebra*, Toronto: Holt, Rinehart and Winston, 1970.

2. Del Grande, J., Duff, G., and Egsgard, J., *Mathematics 12 (Third Edition)*, Toronto: Gage Publishing Ltd., 1980.

3. Dolciani, M., Verman, S., and Wooton, W., *Modern Algebra and Trigonometry*, Don Mills: Thomas Nelson and Sons (Canada) Ltd., 1964.

4. Travers, K., Dalton, L., and Brunner, V., *Using Advanced Algebra*, Toronto: Doubleday Canada Ltd., 1976.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Recognize a polynomial function as a function of the form $f(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_{n-1} x + a_n,\ a_o \neq 0,$ $n \in N$.	
2. Classify a given polynomial function according to (a) degree, and (b) coefficients (integral, rational, or real).	
3. Evaluate an integral polynomial function for a given value of the domain by substitution.	

OBJECTIVES	COMMENTS/ACTIVITIES
4. Divide an integral polynomial in one variable by a binomial of the form (x - a) using the division algorithm.	
5. Divide an integral polynomial in one variable by a binomial of the form (x - a) using synthetic division.	
6. Evaluate an integral polynomial function for a given value of the domain using the Remainder Theorem.	
7. Determine whether a given value of the domain is a zero of a given integral polynomial function.	
8. Determine potential integral zeros of a given integral polynomial function.	
9. Factor a given integral polynomial over the integers.	
10. Determine potential rational zeros of a given rational polynomial function.	
11. Determine the integral zeros of a given integral polynomial function.	
12. Determine the rational zeros of a given rational polynomial function.	

OBJECTIVES	COMMENTS/ACTIVITIES
13. Determine an integral polynomial function given rational zeros.	
14. Determine the possible sums of multiplicities of zeros of a given real polynomial function.	
15. Determine all zeros of a given real polynomial function.	
16. Sketch the graph of a given polynomial function.	

PROBABILITY

There are numerous books which cover this topic, usually together with some statistics. In a short elective component, it is important to just get the feel of calculating some interesting probabilities. If one can experimentally verify or disprove ones' results, so much the better.

OBJECTIVES

The following concepts should be taught as a preamble:

1. The idea of an event E as the outcome of some experiment of trial.

2. The meaning of $P(E)$, the probability of this event occurring. Also the fact that probabilities are numbers between 0 and 1.

3. The concept of independence of events.

4. The meaning of mutually exclusive events.

5. If one has two events A *and* B, then: $P(A \text{ or } B) = P(A) = P(B) - P(A \text{ and } B)$.

COMMENTS/ACTIVITIES

If the above five ideas can be mastered, then one can follow with some simple combinatorial results before some interesting examples can be tried. The student should learn about:

a) Factorial notation

b) that $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$ represents the number of ways of arranging k objects among n locations.

c) the hypergeometric distribution.

One has a box containing N_1 black marbles and N_2 white marbles for a total of $N = N_1 + N_2$ marbles. If one now takes n marbles from the box (at random without replacement) then the probability one obtains n_1 black and n_2 white marbles is $\binom{N_1}{n_1}\binom{n_2}{n_2} \Big/ \binom{N}{n}$

e.g., if the box contains 5 black and 3 white marbles, then in drawing 2 marbles the probability one obtains one of each colour is:

$$\binom{5}{1}\binom{3}{1}\binom{8}{2} = 15/28$$

OBJECTIVES	COMMENTS/ACTIVITIES
continued	With these minimal tools, one can now tackle many interesting problems. For example, when dealt a 5-card hand from a standard deck, following are the poker hands one can obtain: a) nothing b) one pair c) 2 pairs d) 3 of a kind (a triple) e) full house (one triple, one pair) f) four of a kind g) flush (all in one suit) h) straight (5 cards in sequence) i) straight flush (5 cards in sequence in one suit) j) royal flush (highest straight-flush)

If one considers these 10 events as mutually exclusive, one can calculate the probability of each occurring and hence order, via probabilities, the sequence of winning events.

There is a multitude of variations on this one theme alone, namely what happens if you have a wild card? Does this change the order of winning events? All these can be answered, and if in doubt, deal a few hands to check your answer.

Reference:

1. Most of these notations can be found in any respectable text. One in which the card hands are discussed and the probabilities calculated is: *Basic Probability and Applications*, by M. Nosal., Published by W. B. Saunders.

TOPOLOGY

RECOMMENDATIONS: MATH 20, 30, 23, 33.

Reference:

1. *Excursions into Mathematics*, by Beck, Bleicher and Crowe. Published by Worth.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Euler's formula.	Pages 3 - 11.
2. The number of regular polyhedra.	Pages 12 - 16.
3. Tesselation of the plane: Use Euler's formula to show that triangles, hexagons and squares are the only regular polygons one can use to tile a floor.	
4. A European "football" is made of pieces of leather in the shape of regular pentagons and regular hexagons. These are sewed together so that each pentagon is surrounded by hexagons and each hexagon is surrounded (alternately) by three pentagons and three hexagons. Determine the number of pentagons and hexagons of such a football.	
5. Deltahedra: This section deals with non-regular polyhedra, all of whose faces are triangles. It also illustrates methods of constructing these polyhedra out of cardboard.	NOTE: The objectives listed are all fully illustrated in the text with numerous examples that can actually be constructed.
6. If time permits, there is a section of polyhedra without diagonals using Euler's formula.	

TRANSFORMATIONAL GEOMETRY

RECOMMENDATIONS: MATH 20
30, 23, 33

Euclid has been criticized for his use of the super-position argument to prove the familiar side-angle-side (SAS) congruence theorem. Simplified, the argument calls for one to pick up a triangle and attempt to fit it on another. While deficiences have sometimes been pointed out in Euclid's system of axioms, it may be noted that those seeking to evolve more logically satisfactory systems of axioms have generally attempted to by-pass the difficulty.

Although it may be necessary to justify every step of a proof one should not lose the intuitive appeal of Euclid's approach. Most geometry teachers probably explain congruence by placing one triangle over another. E.G., by cutting out cardboard triangles, by tracing one triangle over another, etc.

Basically then, we could consider the result of mapping a figure by some appropriate transformation.

References:

1. Bye, M., Griffiths, T. and Hanwell, A., *Holt Math 4*, Toronto: Holt, Rinehart and Winston, 1980, pages 265-303.

2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: An Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1977, pages 320-344.

3. Ebos, F., and Tuck, B., *Math is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979, pages 353-371.

4. *The Mathematics Teacher (NCTM): Transformation Geometry and the Artwork of M.C. Escher*, December 1976, pages 647-652.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To develop and describe the following different types of transformations:	Graph paper should be used whenever possible.
a) Translations	(For obvious reasons, these are also referred to as slides or glides.)
b) Reflections	A transparent plastic reflector called a MIRA would be an excellent tool in teaching reflections.
c) Rotations	The hands of a clock noting its centre is a very useful example.
d) Dilations	Most students know what similarity (same shape, different size) means. Use examples (photo enlarger, microscope, etc.), to explain dilations.

TRIGONOMETRIC IDENTITIES

Trigonometric equations which are true for all permissible values are called trigonometric identities. There are eight fundamental trigonometric identities:

1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
2. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
3. $\csc\theta = \dfrac{1}{\sin\theta}$
4. $\sec\theta = \dfrac{1}{\cos\theta}$
5. $\tan\theta = \dfrac{1}{\cot\theta}$
6. $\sin^2\theta + \cos^2\theta = 1$
7. $\tan^2\theta + 1 = \sec^2\theta$
8. $1 + \cot^2\theta = \csc^2\theta$

These fundamental identities can be used to develop or prove more complex trigonometric identities. To prove a trigonometric identity one should adopt the following objectives.

References:

1. Travers, K.J., Dalton, L.C., Brunner, V.F., *Using Advanced Algebra*, Toronto: Doubleday Canada Ltd., 1975.

2. Johnson, R.E., Lendsey, L.L., Slesnick, W.E., Bates, G.E., *Algebra and Trigonometry*, London: Addison-Wesley Publishing Co., 1976.

3. Dottori, D., Knill, G., Stewart, J., *Foundations of Mathematics for Tomorrow - Senior SI Metric Edition*, Toronto: McGraw-Hill Ryerson Ltd., 1978.

4. Del Grande, J.J., Duff, G.F.D., Egsgard, J.C., *Mathematics 12 - Third Edition*, Toronto: Gage Publishing Ltd., 1980.

5. Welchons, A.M., Krickenberger, W.R., *Trigonometry With Tables*, Toronto: Ginn and Company, 1957.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use fundamental identities to develop or prove more complex trigonometric identities.	Work with each side separately. Substitute fundamental or proven identities into both sides. Do any subtraction, addition, multiplication, division or simplification that may be necessary. Try to get each side into identical format and hence prove the identity.

VECTORS

Vectors provide a useful tool for analyzing problems that deal with trips, forces, velocities, accelerations and displacement. A vector is a quantity that requires both a magnitude and direction to describe it.

References:

1. Bye, M. and Elliott, H., *Math Probe 3,* Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.

2. Burns, A.G. Pinkney, R.G., and Del Grande, J.J., *Mathematics for a Modern World: Book 3,* Toronto, Gage Educational Publishing Ltd., 1976.

3. Dottori, D., Knill, G. and Seymour, J., *Applied Mathematics for Today: Intermediate,* Toronto: McGraw-Hill Ryerson Ltd., 1976.

4. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Introduction,* Toronto: McGraw-Hill Ryerson Ltd., 1974. Nichole, *Modern Intermediate Algebra.*

5. Dottori, D., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Intermediate,* Toronto: McGraw-Hill Ryerson Ltd., 1978.

6. Ebos, F., and Tuck, B., *Math is 4,* Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

7. Hilton, A., *Vectors,* London: Macdonald Educational Colour Units, 1976.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Understand those phenomena which possess magnitude and direction.	Describe five situations in which vectors are used.
2. Add and subtract vectors.	Create and solve problems using vectors.
3. Multiply a vector by a scalar.	
4. Solve problems using vectors.	

ADDITIONAL COMMENTS

Alberta Education would welcome any comments and suggestions pertaining to the Mathematics 30/33 Interim Curriculum Guide. Please contact or forward comments to the Regional Office Mathematics Consultant in your area.

METRICATION POLICY

It is the policy of Alberta Education that:

1. SI units become the principal system of measurement in the curriculum of the schools in the province;
2. the change to the use of SI units in schools be such that the instructional programs are predominantly metric by June, 1978;
3. changes in the curriculum of individual schools proceed in concert with corresponding changes in industry and commerce;
4. selected consultative and in-service resources be made available to teachers for professional preparation in the integration of SI units in their instructional programs;
5. conversion of the schools to the metric system (SI) be carried out on a pre-planned basis with a gradual replacement and/or modification of measurement-sensitive resources;
6. conversion costs will generally be borne by the responsibility centre incurring them;
7. conversion will be accomplished by means of existing administrative structures, and there will be only a minimum number of short-term special purpose assignments associated with the change to SI.

EXPLICATION OF THE METRICATION POLICY

1 *. . . that SI units become the principal system of measurement in the curriculum of the schools in the province;*

1.1 As part of a world-wide movement to standardize commonly used elements of trade, Canada has committed itself to the use of SI units by the early 1980's. It is already evident that many sections of our society which use measures will be predominantly metric before 1980.

It has become imperative that the school curriculum prepare students to cope with metric measures in all facets of their life. In addition many students will require a detailed knowledge of more specilized units used in specific career fields.

1.2 Some imperial units will be in use for some time; therefore as teachers introduce SI they are expected to retain selected reference to imperial units. This teaching of specific imperial units should be related only to those that are relevant to student needs and should be kept to a minimum. Mathematical conversions from one system to the other are to be avoided wherever possible.

1.3 Resource materials for the classroom should use SI units as the principal measuring system with only such references to the imperial or "old" metric systems as are unavoidable.

2 *. . . that the change to the use of SI units in schools be such that the instructional programs are predominantly metric by June, 1978.*

2.1 In a predominantly metric program the student, depending upon age and ability, is familiar with the appropriate metric units of length, area, volume, mass and temperature, and uses these units in school for making and expressing measurements in all areas of the curriculum.

2.2 The date (June, 1978) is a general target by which it is expected that curriculum guides and programs of studies and the various acts and regulations will have been changed to reflect the new measurement language. Most schools will have begun the transition of their classroom programs by June, 1978; indeed many schools will have completed the change before this date.

2.3 It is entirely possible that some of the high school technical and vocational programs will have a need to continue teaching the use of imperial units. It may take a specific technological area some considerable time to make the change, especially one that uses machinery with a long working life.

2.4 The correct usage of the SI units will be directed by the **Metric Practice Guide** (CSA Z234.1 - 1973) or the **Metric Style Guide** (Council of Ministers of Education, Canada). A copy of the **Metric Style Guide** has been made available to each teacher (in English or French).

3 *. . . that changes in the curriculum of individual schools proceed in concert with corresponding changes in industry and commerce.*

3.1 The Canadian Metric Commission under the federal Ministry of Industry, Trade and Commerce has been coordinating the change to metrics since 1970. Timelines and metrication dates have been established for many sectors of the economy. Educators should keep themselves informed as to progress in both the

technical areas in which they may be teaching and the activities occurring in the community.

3.2 The Department of Education will attempt to keep schools informed regarding progress in the use of metric units.

4 *. . . that selected consultative and inservice resources be made available to teachers for professional preparation in the integration of SI units in their instructional programs.*

4.1 Achieving metric conversion by the target date is dependent upon many factors, not the least of which is the whole-hearted cooperation by educators in carrying out their assigned roles. In preparing teachers and administrators to better cope with the changes, the Department of Education will provide consultative and inservice resources through the Regional Offices in Grande Prairie, Edmonton, Red Deer, Calgary and Lethbridge.

4.2 The staff of a school should plan co-operatively to bring a unified approach to the teaching and use of measurement. Metrication goes beyond the formal content of the curriculum. The changeover with its many implications will affect everyone associated with the process of education.

4.3 Since measurement is an activity related process, it follows that individuals — teachers as well as students — learn the metric units while measuring. That is, some degree of active involvement in measuring or in using the units that one is learning.

4.4 Very few people will have to know the entire metric system. That is, when presenting SI, one should only attempt to deal with that part of the system which is necessary for the task at hand.

5 *. . . that conversion of school programs to SI be carried out on a pre-planned basis with a gradual replacement and/or modification of measurement-sensitive resources.*

5.1 Most measurement-sensitive devices will have to be replaced over the next few years. Items ranging from inexpensive rulers to costly metal lathes will have to be either replaced or modified. Exactly when and how much modification versus replacement occurs depends upon the economics. For a machine that is near the end of its useful life, any modifications will have to be minimal. For a nearly new machine a scale replacement or recalibration may be the best course of action. In any event, common sense must prevail.

5.2 Any new acquisitions of machines or tools should be those with metric capabilities. In spite of the best planning and careful budgeting, there is bound to be some frustration as suppliers fail to deliver as promised and as people do not respond as predicted.

6 *. . . that conversion costs will generally be borne by the responsibility centre incurring them.*

6.1 The term "responsibility centre" is meant to refer to those parts of the administrative structure which are responsible for budgeting and purchasing services, materials, and equipment for schools.

6.2 By placing responsibility for costs as close as possible to the actual use of goods or services, it is hoped that there will be a greater accountability in the changing over to metrics.

6.3 Implicit in this policy is the idea that those exercising responsibility will do so with restraint.

7 *. . . that conversion will be accomplished by means of existing administrative structures, and there will only be a minimum number of short-term special-purpose assignments associated with the change to SI.*

7.1 In other words, as far as is possible, there will be only temporary positions associated with metrication. In addition, administrative costs are to be kept as low as possible, with the additional work load being handled on a contract or short-term assignment basis. In this way, it is hoped that the money spent on metrication will have maximum effect on the classroom.

Strategies of Problem Solving

In the teaching/learning of problem solving, an instructional approach should be used which helps students learn and choose procedures for solving problems. These procedures are easy to state and recognize, but they are often quite elusive when teaching. Difficulty frequently exists when teaching problem solving because, unlike the teaching of computational skills or concepts, there is no specific content involved. In problem solving, an individually acquired set of processes is brought to bear on a situation that confronts the individual.

There are generally four procedures (steps) which appear inherent in problem solving. These procedures, their descriptions and associated strategies have been compiled and adapted from a variety of sources and authors (George Polya, J.F. LeBlanc, Ohio Department of Education, Math Resource Project, 1980, NCTM Yearbook) and are listed below:

STEPS IN PROBLEM SOLVING

1) UNDERSTAND THE PROBLEM

What is the problem? What are you trying to find? What is happening? What are you asked to do?

Suggested Strategies:

- Paraphrase the problem or question (Restate the problem in your own words to internalize what the problem entails.)

- Identify wanted, given and needed information (Helps students focus on what is yet to be determined from problem statement as well as listing information so that they may better be able to discover a relationship between what is known and what is required.)

- Make a drawing (May help to depict the information of a problem, especially situations involving geometric ideas.)

- Act it out (Helps to picture how the problem actions occur and how they are related thereby giving a better understanding of the problem.)

- Check for hidden assumptions (What precisely does the problem say or not say? Are you assuming something that may not be implied? Beware of mistaken inferences.)

2) DEVISE A PLAN TO SOLVE THE PROBLEM

What operations should you use? What do you need to do to solve the problem? How can you obtain more information or data to seek the solution?

Suggested Strategies:

- Solve a simpler (or similar) problem (Momentarily set aside the original problem to work on a simpler or similar case. Hopefully the relationship of the simpler problem will point to the solution for the original problem.)

- Construct a table (Organizing data in tabular form makes it easier to establish patterns and to identify information which is missing.)

- Look for a pattern or trend (Does a pattern continue or exist? In connection with the use of a table, graph, etc., patterns or trends may be more apparent.)

- Solve part of the problem (Sometimes a series of actions each dependent upon the preceding one, is required to reach a solution. Similarly it may be that certain initial actions will either produce a solution or uncover additional information to simplify the task of solving the problem.)

- Make a graph or numberline (May help organize information in such a way that it makes the relationship between given information and desired solution more apparent.)

- Make a diagram or model (When using the model strategy attempt to select objects or actions to model those from the actual problem that represents the situation accurately and enables you to relate the simplified problem to the actual problem. May be used in connection with, or in place of other similar strategies, i.e., acting out the problem.)

- Guess and check (Guessing for a solution should not be associated with aimless casting about for an answer. The key element to this strategy is the "and check" when the problem solver checks his guesses against the problem conditions to determine how to improve his guess. This process is repeated until the answer appears reasonable. An advantage of this "guess and check" strategy is that it gets the individual involved in finding a solution by establishing a starting point from which he can progress. Used constructively with a table or graph this strategy may be a valuable tool.)

- Work backwards (Frequently, problems are posed in which the final conditions of an action are given and a condition is asked for which occurred earlier or which caused the final outcome. Under these circumstances working backwards may be valuable.)

- Change your point of view (Some problems require a different point of view to be taken. Often one tends to have a "mind set" or certain perspective of the problem which creates a difficulty in discovering a solution. Frequently, if the first plan adopted is not successful, the tendency is to return to the same point of view and adopt a new plan. This may be productive, but might also result in continuous failure to obtain a solution. Attempt to discard previous notions of the problem and try to redefine the problem in a completely different way.)

- Write an open sentence or equation (Often in conjunction with other strategies - using a table, diagram, etc., one selects appropriate notation and attempts to represent a relationship between given and sought information in an open sentence.)

3. CARRY OUT THE PLAN

For some students the strategy/strategies selected may not lend itself/themselves to a solution. If the plan does not work, the problem solver should revise the plan, review step 1, and/or try another plan or combination of plans from step 2.

4. LOOK BACK AT THE STEPS TAKEN (Consolidating Gains)

Is the result reasonable and correct? Is there another method of solution? Is there another solution? Is obtaining the answer the end of the problem?

- Generalize (Obtaining an answer is not necessarily the end of a problem. Re-examination of the problem, the result and the way it was obtained will frequently generate insights far more significant than the answer to the specific situation. It may enable the student to solve whole classes of similar and even more difficult problems.)

- Check the solution (The very length of a problem or the fact that symbolic notation is used may tend to make one lose sight of the original problem. Does the answer appear reasonable, does it satisfy all the problem requirements?)

- Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem solving is to study the processes that lead to solutions to problems. Once a solution is discovered, search the problem for further insights and unsuspected ideas and relationships.)

- Find another solution (Students tend to approach many problem situations with the expectation of only one correct solution. In many practical, daily life situations there may be many answers that are correct and acceptable.

- Study the solution process (Studying the process of solution makes the activity of problem solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.

It must be noted that the four steps of the above model are not necessarily discreet. For example, one may move without notice into Step 2 while attempting to generate more information to understand the problem better.

If the 4-step model is used, the key is to select an appropriate strategy or strategies to help answer the questions suggested by each step. The strategies listed, and those devised by students will hopefully alter the problem information, organize it, expand it, and make it more easily understood. Strategies then may be thought of as the tools of problem solving and the 4-step model, the blueprint.

Recommendations for School Mathematics of the 1980s

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The National Council of Teachers of Mathematics recommends that -

1. problem solving be the focus of school mathematics in the 1980's;

2. basic skills in mathematics be defined to encompass more than computational facility;

3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;

5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;

6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;

7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;

8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.